Footbridge Pedestrian Vibration Limits

Part 3: Background to Response Calculation

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Summary

The UK Highways Agency has commissioned two companion studies for the review of the dynamic sensitivity of footbridges. As part of their contribution to this work, Flint & Neill Partnership have undertaken a fairly extensive theoretical review of the combined subjects of pedestrian loading, response prediction and user perception of bridge motion. The results of this study form the basis for a new approach to the assessment of the dynamic responses of footbridges, and aspects of which are being presented in a related papers at this conference [1][2].

This paper presents of some of the background to the development the design strategy developed for the prediction of vertical responses. The material presented in this paper serves two purposes (a) it provides useful background in order to assist practicing engineers to better understand a few of the many important theoretical issues, and (b) it provides an example of a simple analysis algorithm (able to be implemented in a spreadsheet) that, in many cases, will enable engineers to calculate pedestrian dynamic responses with the same precision as that of a full time history analysis.

(This is one of a series of papers [3][4][5][6] *written to report on the progress of the above study. Two companion papers* [1][2] *are being presented at this conference that deal principally with the development of proposals for more relevant and searching assessment rules.)*

Keywords: Footbridge; dynamic properties; pedestrian-induced vibrations; natural frequency; damping; stochastic analysis, spectral leakage

1. Introduction

The main content of this paper deals with 3 specific issues:-

- A description of the development of factors to cater for the relative nuisance value of pedestrian groups.;
- A discussion of some particular problems that occur in the prediction of vertical bridge responses due to crowd loading using spectral analytical methods;
- A method for the calculation of dynamic responses at resonance.

2. The relative nuisance value of pedestrian groups

2.1 A definition for the 'relative nuisance' value of the response

BS6472 [7] introduces the notion of vibration doses. In this a useful equivalence is made between rare relatively large amplitude motion, and more frequent response of a smaller magnitude. The vibration dose time-dependency rule adopted means that a two-fold increase in vibration amplitude is equivalent to a 16 fold decrease in the duration of the vibration. (In other words a fourth power time dependency rule.)

Such a relationship between the amplitude and duration of pedestrian response is thought to be a more appropriate measure of the nuisance (comfort) level of bridge response than either the simple RMS or the maximum value of the signal. In particular this approach provides the means to compare,

- slowly decaying, lightly damped response patterns with the obviously lower nuisance level of a highly damped response having the same peak amplitude,
- simple sinusoidal signals typical of the passage of a single pedestrian with probabilistic extreme value expressions of response that might result from a spectral analysis.

Hence we choose to start this section by defining that the relative nuisance level of a given response pattern is to be determined by means of the 'root-root-4th power mean' (RR4M) of the signal.

2.2 Pedestrian groups

While it is recognised that it is possible for the members of small groups of pedestrians to walk in step with each other this is not normally what occurs in most cases. Observations by TRRL [8] on 1200 people walking normally over footbridges has produced the result that 4 per cent of the sample observations were pairs of people walking together, in step, for at least ten paces. One such group of three was observed, but larger groups were not in step. However these results, rather than indicating a tendency that pedestrians might be commonly walk in step, seems to demonstrate that synchronisation normally occurs with a similar likelihood to that of wholly random occurrences.

In Figure 1 below we show a simple example representative of two pedestrians walking in step, of two superimposed sinusoidal signals with slightly different frequencies. This demonstrates the expected beating of the net response as the component signals match periodically.

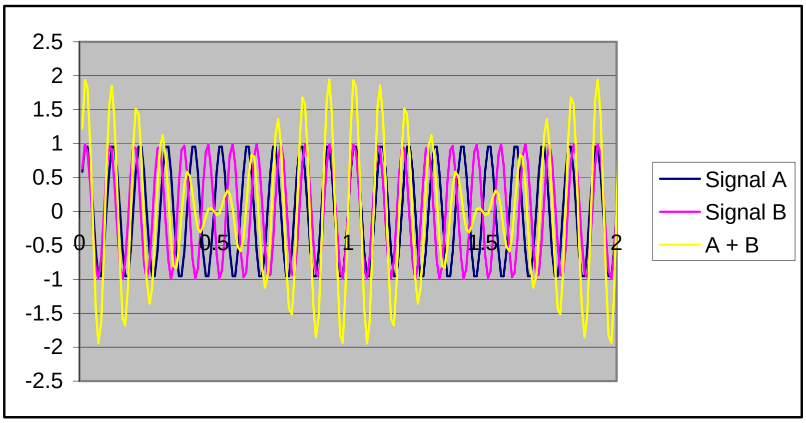

Figure 1: The beating net response of 2 similar signals combined

Given that normally there is only a relatively small variation in the pace frequency between different pedestrians, pairs of pedestrians will appear to be walking in step quite often even though they may not remain so on average. In the case of larger groups it is evident that the pedestrian group as a whole will 'appear to be in step' for a smaller proportion of the time, and that the mean time between maximum beats gets longer. In crowded conditions the extreme maxima become very rare occurrences and are normally described by a probabilistic model.

For the purposes of this study we assume that normally pedestrians in groups do not deliberately walk in step. Thus in order to investigate the effect of small groups we assume that the pedestrians in the group,

- cross the bridge together, so that at any point in time they are at the same location as each other,
- have a similar mass as each other and apply a dynamic load of a similar magnitude, but that they
- are uncorrelated, their pace frequency and timing being determined randomly from simple distribution functions.

At any point in time the response produced by a single pedestrian is close to sinusoidal, and the contributions made by uncorrelated pedestrian passages to the net response tend to have the same magnitude, a similar but different frequency, and a different phase. We can therefore examine the average effect of combinations of pedestrians by considering the mean amplitude of randomly combinations of sine waves.

Matsumoto [9] and others previously used the RMS of the response as their measure of relative discomfort and arrived at the conclusion that the mean RMS response is proportional to ÖN, where N is the number in the group.

By the use of convolution integrals it is a fairly straightforward matter to verify this result. For example the integral shown below determines the average of the RMS response for every possible combination of 3 signals of amplitude a.

$$RMS_{N=3}=\left[\frac{1}{(2.p)^3}.\int_0^{2.p}\int_0^{2.p}\int_0^{2.p}(a.\sin(x)+a.\sin(y)+a.\sin(z))^2\,dx.dy.dz\right]^{0.5}=1.225a \qquad (1)$$

And results of other similar equations can easily be generalised to the aforementioned $ÖN$ result,

$$RMS_N=RMS_1.[N]^{0.5} \qquad \text{(Where, } RMS_1=[0.5]^{0.5}\text{)} \qquad (2)$$

In a very similar manner, using our revised measure of discomfort, the integral of the 'root-root-4th power mean' (*RR*4*M*) can be obtained from,

$$RR4M_{N=3}=\left[\frac{1}{(2.p)^3}.\int_0^{2.p}\int_0^{2.p}\int_0^{2.p}(a.\sin(x)+a.\sin(y)+a.\sin(z))^4\,dx.dy.dz\right]^{0.25}=1.54a \qquad (3)$$

Although not quite as simple as equation (2) above, the result of the 'root-root 4th power mean' can still be generalised to a manageable form, as follows,

$$RR4M_N=RR4M_1.\left[\sum_{i=1}^{N}(1+4.(i-1))\right]^{0.25} \qquad \text{(Where, } RR4M_1=\left[\frac{3}{8}\right]^{0.25}\text{)} \qquad (4)$$

Thus for small groups of pedestrians we are able to express the average relative increase in nuisance level with respect to that of a single pedestrian using equation (4) above

$$k_g(N)=RR4M_N\big/RR4M_1 \qquad (5)$$

Values obtained from (5) are given below and compared with the increase in RMS predicted by the $ÖN$ model.

Table 1: Alternative measures of group responses

Number in group (N)	*RMS/a* (std.dev.)	*RR4M/a*	*RR4M* / *RMS*	Relative increase in *RMS*	Relative increase in Discomfort	**1.188**ÖN
1	0.71	0.78	1.107	1.00	1.00	1.19
2	1.00	1.22	1.225	1.41	1.57	1.68
3	1.22	1.54	1.257	1.73	1.97	2.06
4	1.41	1.80	1.273	2.00	2.30	2.38
5	1.58	2.03	1.282	2.24	2.59	2.66
6	1.73	2.23	1.288	2.45	2.85	2.91
7	1.87	2.42	1.292	2.65	3.09	3.14
8	2.00	2.59	1.295	2.83	3.31	3.36
9	2.12	2.75	1.297	3.00	3.52	3.56
10	2.24	2.91	1.299	3.16	3.71	3.76
20	3.16	4.14	1.308	4.47	5.28	5.31
50	5.00	6.56	1.313	7.07	8.39	8.40
100	7.07	9.29	1.314	10.00	11.88	11.88
500	15.81	20.80	1.316	22.36	26.58	26.56

It is shown that 1.188$ÖN$ can be used to give a very good approximation to the full expression for discomfort for groups >3, and the 20% over-estimate it gives for single pedestrians is normally of no importance since the response due to a single pedestrian is never likely to be a real design case.

For completeness, it is worth noting here that the $ÖN$ model only applies to the prediction of RMS values and its use does not give response maxima (for which a value of N must be used).

2.3 Crowds

In the extreme, responses due pedestrian loading (or for that matter wind) can only be expressed stochastically. Usually the probability of the amplitude (here of the deck vertical acceleration) having a particular value at any instant in time can be taken to be well modelled by a normal distribution. When we integrate values from a normal distribution and examine the ratio of the RR4M to the RMS we find that RR4M/RMS = 1.316, this is in exact agreement with the results for groups given previously in table 1.

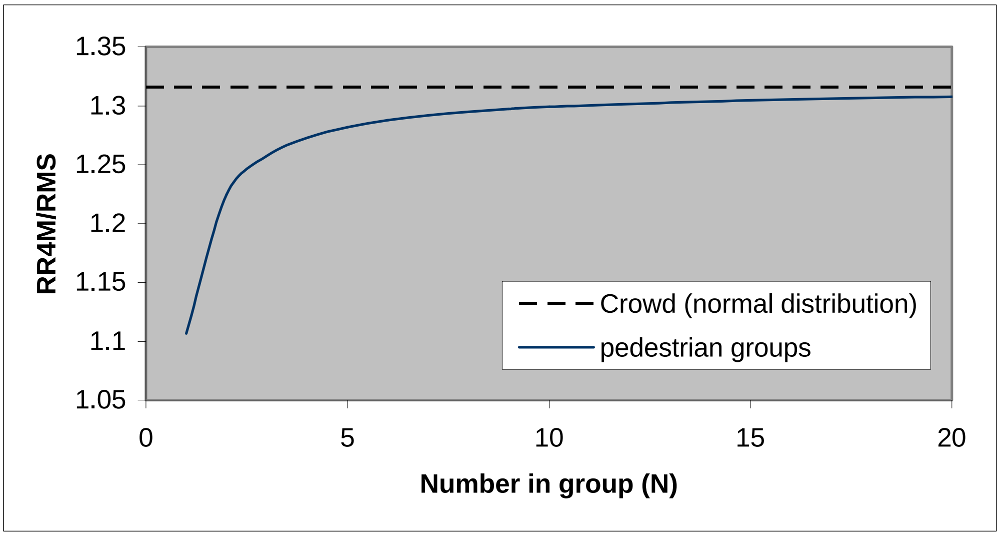

Figure 2: Convergence of the 4th power mean nuisance measure for large crowds

3. On the calculation of footbridge responses due to crowd loading

(Why steady state spectral models do not always work)

The second part of this paper discusses several problems that arise in the use of spectral analytical methods to the prediction of bridge vertical responses to pedestrian excitation. The subject matter presented here concerns the validity of the signal processing techniques at the heart of spectral analysis, and should be of particular interest to those readers who have previously understood that pedestrian loading is easily modelled as a stationary random process.

Both Matsumoto in 1978 and more recently Brownjohn [10] and others have stated that the spectral contribution to the loading at a particular frequency can be treated as a near steady state condition. We can easily demonstrate what this means in practice, without needing to discuss the nature of spectra, by simply recycling a single pedestrian across the span over and over till a near steady condition is achieved.

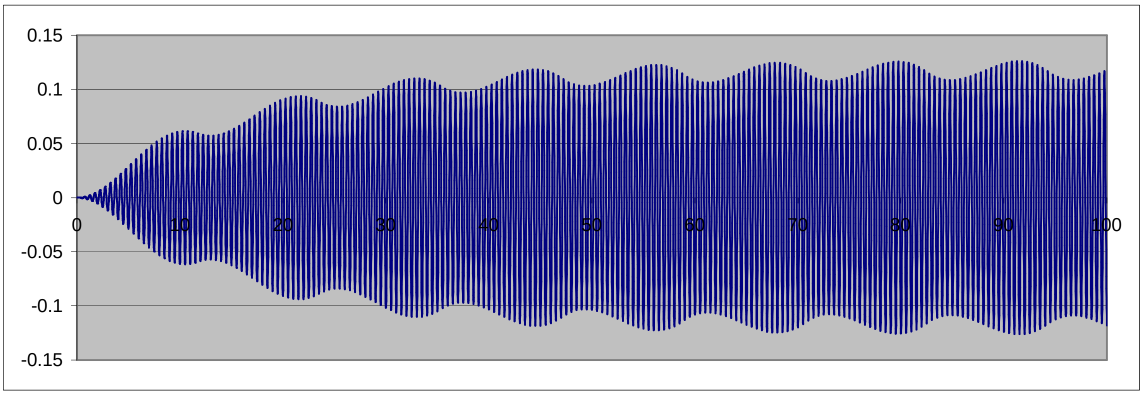

Figure 3: Response for a recycling pedestrian load at resonance, pace frequency = 2.1Hz

Figure 3 illustrates how the amplitude of bridge response grows with each successive crossing. Eventually, after some time, near steady conditions prevail and it can be demonstrated that the response magnification Q that occurs is that of a simple single degree of freedom system.

For a simple 1 degree of freedom dynamic system the steady state response amplitude due to a load applied with a frequency is $A(w)=Q.(F_0/K)$ where,

$$Q(w)=\frac{1}{\sqrt{\left[1-\left(\frac{w}{w_n}\right)^2\right]^2+\left[2x.\left(\frac{w}{w_n}\right)\right]^2}} \qquad (6)$$

However by making one small change to the response model we can get a very different result. The change is simply to restart each subsequent crossing with a randomly timed first step. In other words the initial phase angle of the applied is randomly reset at the start of each successive pedestrian crossing. Even though idealised load models may assume that each pedestrian remains at a constant frequency as they cross the span, there is no reason why successive pedestrian should remain in step. The effect that this has on the calculated response is shown in figure 4 below.

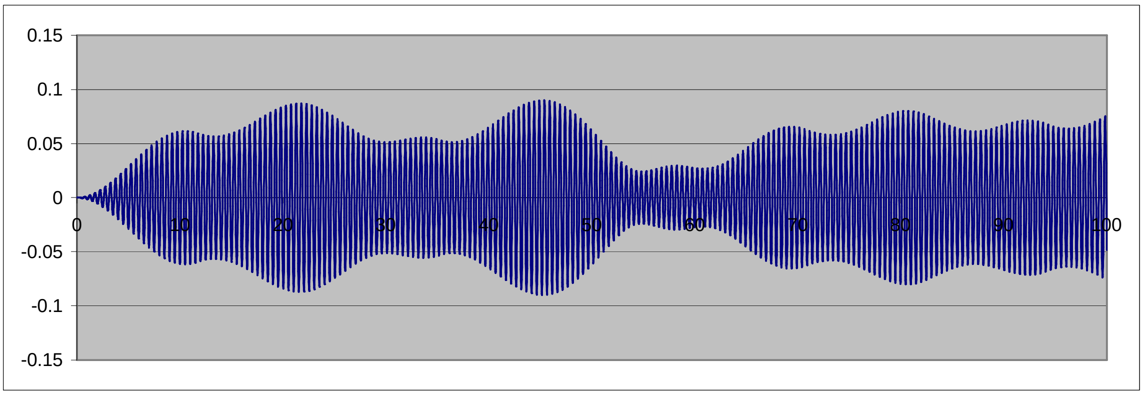

Figure 4 Response for a recycling pedestrian load at resonance, resetting starting phase, pace frequency = 2.1Hz

Clearly the response is now, (a) more unsteady and (b) generally smaller. Over a longer timescale, occasionally the peak response will still reach the same amplitudes as the steady state model, but only very rarely.

However if we now perform a Fourier analysis on the applied load we can see that instead of the input energy all being applied at the pace frequency as expected, it appears to have 'leaked' into the adjacent frequencies.

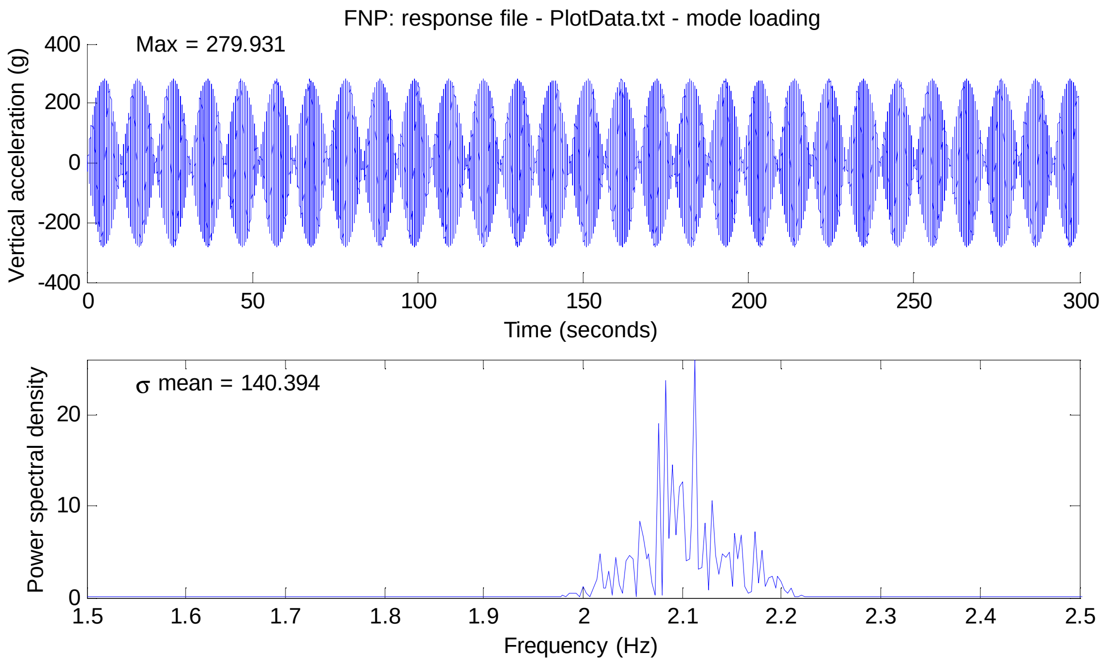

Figure 5: Leakage of the applied load into the adjacent frequencies, pace frequency = 2.1Hz

This phenomenon of spectral leakage is well known (at least among signal processing engineers) and occurs in our approximation of the pedestrian loading in at least 3 places

- In the lack of phase correlation between one pedestrian and the next (described above).
- In the variations and imperfections of individual walking excitation (described by Brownjohn [10]).
- As a result of long beat frequencies between the pace frequency and the span length (where in more complex mode shapes the pedestrian is in phase with the motion near the mode maxima but out of phase at anti-nodes).

However the net effect that this has on calculated responses remains unclear, because while the response due to a pedestrian walking at the mode frequency is always reduced, leakage can also act to increase responses at other frequencies. An indication of the effect this can have on the calculated response is given in Figure 6 below.

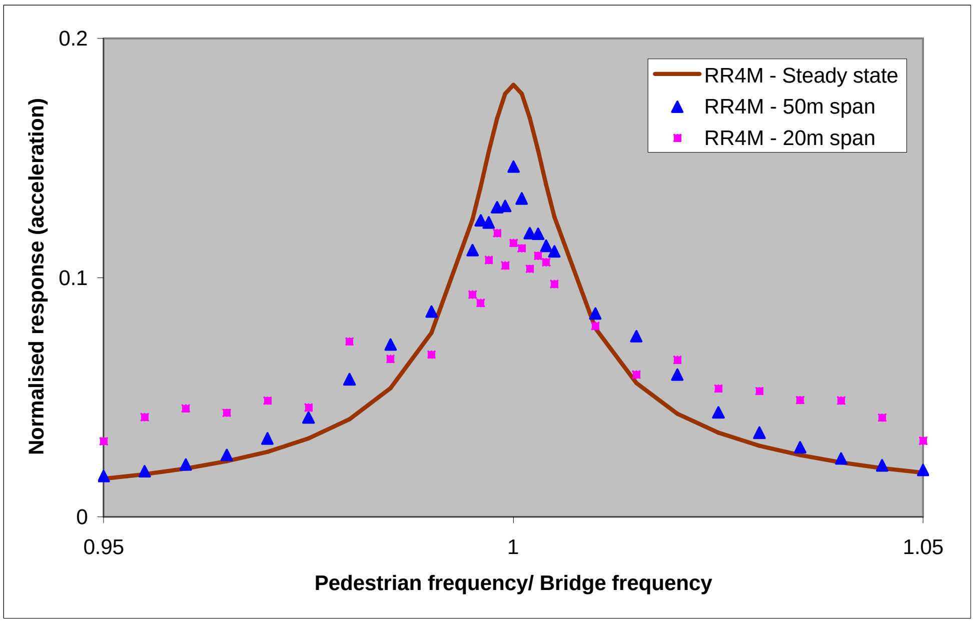

Figure 6: Effect of spectral leakage on calculated responses around resonance

There is nothing in what has been said that invalidates the spectral analytical approach, but it becomes apparent that a complete treatment of this subject is more complicated than it initially appears. All of the above can easily and adequately be dealt with by a little widening of the assumed load spectrum and a slightly increased safety factor. However if nothing else, this example shows that it is important that engineers should not get too carried away refining the load model which in addition to the above will vary greatly from site to site and time to time.

4. On the calculation of dynamic responses at resonance

BD 37/01 [11] describes the need to estimate the dynamic response due to a simple fluctuating vertical point load, *F*, that moves across the span of the bridge at a constant speed v_t

$F = 180.\sin(2.f_o.t)$ (in N), where t is the time (in sec) (7)

$v_t = 0.9\, f_o$ (in m/sec) (8)

Two approaches are provided to achieve this end, the first is to apply some simplified rules based pre-calculated results for simple spans, the second is to perform an explicit dynamic analysis.

Simplified response models

In many if not most instances these models are simply not accurate, especially in the hands of the inexperienced.

Explicit dynamic analysis

In general the calculation of dynamic responses is considered to be a specialist task requiring the use of dedicated dynamic software. Experience suggests that when many engineering practices need to calculate the dynamic response using the explicit method this is considered to be a complex and somewhat difficult process.

The third way

The purpose of this section of the paper is to point out that there is an extremely simple way of calculating dynamic responses at resonance, without any error and without the need to use specialist software. Furthermore this calculation method is so simple that there should never be any need to resort to the use of other less accurate rules such as that contained in BD37/01.

Theory

When excited at resonance because the excitation frequency and free decay frequencies are matched the response history is always of the form of a simple signal at the natural frequency f_o , whose amplitude is modulated with time.

At any given moment in time the response is influenced by two terms,

- the work done by the applied force in exciting the motion, and
- the energy lost over each cycle by the damping.

For a simple mass being excited by a periodic force at resonance,

$$F = F_0.a.\sin(w.t) \tag{9}$$

the response can be described by,

$$x(t) = a(t).\sin(w.t) \tag{10}$$

and the work done by this force over one cycle by,

$$WD = \int_0^{\frac{2.p}{w}} F.\dot{x}.dt \tag{11}$$

If we treat $a(t)$ as being constant for the moment we can integrate the rate of doing work (the input power) as a function of the local amplitude.

$$WD/\sec = \frac{w.a.F_0}{2} \tag{12}$$

Now this results in an increase in energy in the system that can be expressed as

$$E + \Delta E = \frac{1}{2}.M.(\dot{x} + \Delta\dot{x})^2 \tag{13}$$

Which, for a small change in energy gives

$$\partial E = M.w^2.a.\partial a \tag{14}$$

By equating (12) and (14) we obtain an expression for the rate of increase in amplitude caused by the force F_0,

$$\partial a = \frac{1}{2}.\frac{F_0}{K}.w.\partial t \tag{15}$$

For low damping the logarithmic decrement is approximately equal to the fractional decrease in amplitude during one cycle of vibration.

$$d \approx \frac{\Delta a}{a} \tag{16}$$

Rewriting this to express the loss of amplitude per unit time, and using damping ratio instead of log.dec. gives,

$$\partial a = -a x.w.\partial t \tag{17}$$

Hence we can now write a simple expression for the net rate of change in amplitude at any moment in time,

$$\partial a = \left(\frac{1}{2}.\frac{F_0}{K} - a.x\right)w.\partial t \tag{18}$$

Where $w^2=K/M$; K and M are respectively the generalised stiffness and mass of the mode under investigation.

Application

Simply rewrite equation (10) as a recurrence formula. And for suitably small time steps (say $\Delta t=0.001$ sec), this can be used to determine the bounding amplitude at each moment in time.

$$a_{t+\Delta t} = a_t + \left(\frac{1}{2}.\frac{F_0(t)}{K} - a_t.x\right)w.\Delta t \tag{19}$$

If required the amplitude time history at any moment in time can be obtained by modulating the bounding amplitude for $a(t)$ using equation (10) above.

F_0 is written as a function of t because the effective force applied to a structure also depends on a mode influence coefficient determined from where the load is applied within the span. For a fluctuating force moving with a constant speed V across a span s,

$$F_0(t) = F.g\left(\frac{V.t}{s}.p\right) \quad \text{and} \quad F_0(t) = 0 \text{ if } V.t > s \tag{20}$$

Where $?(x)$ is the normalised mode shape that corresponds to K and M above.

While for a simple sinusoidal mode shape,

$$F_0(t) = F.\sin\left(\frac{V.t}{s}.p\right) \quad \text{and} \quad F_0(t) = 0 \text{ if } V.t > s \tag{21}$$

These equations can easily be implemented in a spreadsheet and require no specialist software at all.

Figure 7 below compares the use of formula (19) in a spreadsheet with the results of a full time history analysis using Duhamel integration (which is itself not especially difficult).

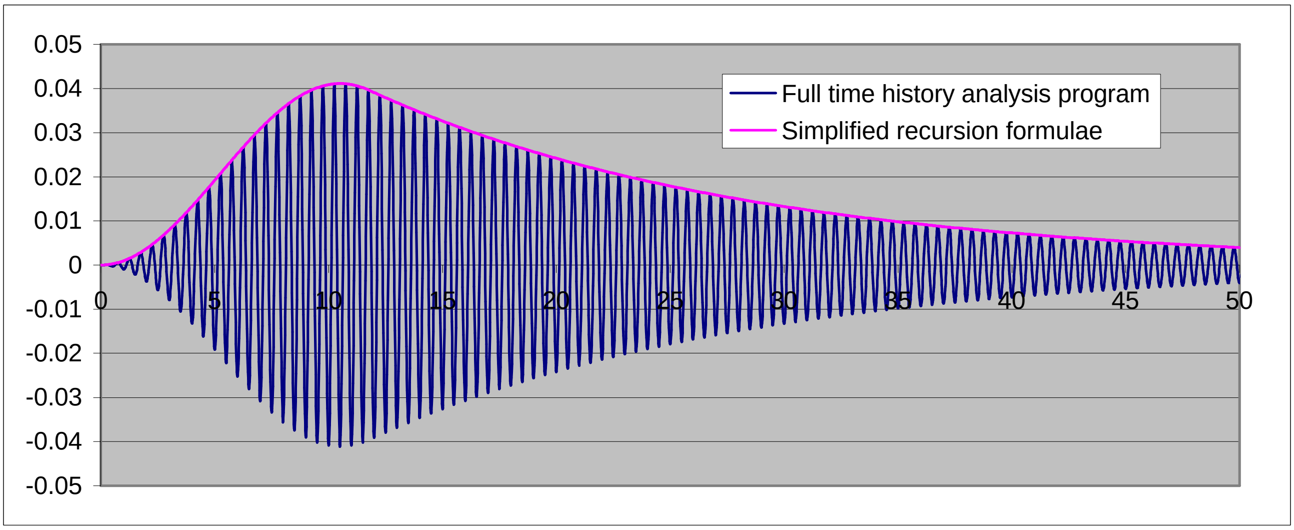

Figure 7: Comparison of full dynamic analysis with simple recursion formula

The effectiveness of the routine is demonstrated by its ability to calculate values for the dynamic response factor ψ of BD37/01 to allow for the combined effects of damping and span length.

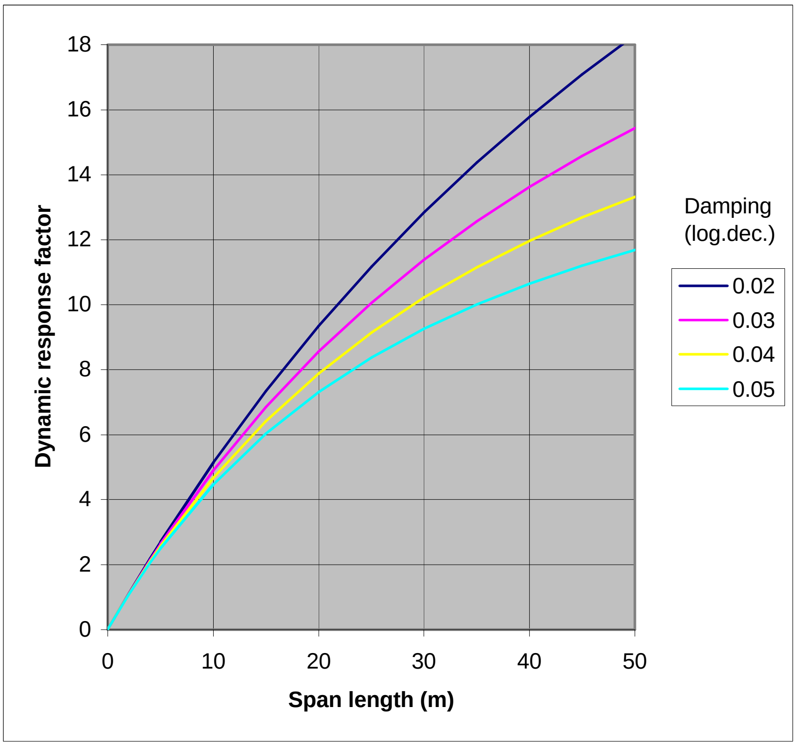

Figure 8: Dynamic response factor y

With this intermediate approach at our disposal, that can easily calculate responses for any shape of mode, there ought to be no need to use approximate configuration factors from documents such as BD37/01 (which are can still be difficult to apply because of the need to approximate the input), instead responses can always be calculated precisely.

Programming

As an example of the ease with which this method can be applied there follows a sample program that reproduces the values in Figure 8 above.

```
Function BD37xi(p, dr, L)
   BD37xi = 180 / 700 * netresp(p, dr, 1, 1, L)
End Function

Function netresp(p, dr, F0, k, L)
   Pi = 3.14159
   vel = 0.9 * (p / 2 / Pi)
   tmax = L / vel
   tstep = 0.01
   disp = 0
   netresp = 0
   For t = 0 To tmax Step tstep
      x = vel * t
      effectiveF0 = F0 * Sin(x / L * Pi)
      disp = disp + respdx(disp, tstep, p, dr, effectiveF0, k)
      If (Abs(disp) > netresp) Then netresp = Abs(disp)
   Next t
End Function

Function respdx(x, dt, p, dr, F0, k)
   If F0 = 0 Then F0 = 0.00001
   respdx = dt * p * (F0 / k) / 2 * (1 - 2 * x * dr / (F0 / k))
End Function
```

5. Conclusions

- This document uses a more consistent approach to the assessment of the relative nuisance value of pedestrian groups than is generally in use at the present time by means of the use of the 'root-root-4th power mean' of the acceleration signal. The relative nuisance value for different sized pedestrian groups is determined, and it is demonstrated that this technique provides consistent results when applied to large fully random populations.
- The phenomenon of spectral leakage is encountered and it is demonstrated that pedestrian loading is not really a stationary random process after all, but that with caution it can still be treated as such.
- A simple analysis algorithm (able to be implemented in a spreadsheet) is presented that, in many cases, will enable engineers to calculate pedestrian dynamic responses with the same precision as a full time history analysis.

6. A cautionary footnote

At times pedestrians do walk in step and we cannot pretend otherwise, for example walking down a slope or stairs will often result in different frequencies and greater synchronisation.

When predictive analyses agree closely with tests (particularly crowd tests) these should be considered fortuitous and not evidence that the model used was a good one (at least not without considerable forensic effort).

It is important that we keep our feet on the ground at all times, protecting safety issues where necessary with a healthy margin. However we should try not to get carried and necessarily disapprove of lively bridges when safety is not a concern.

7. References

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